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Generation of Magnetic Gradients for Biological Systems Stimulation: Mathematical and Geometric Characterization

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Abstract. Mathematical and geometric characterization for the magnetic field generated in three different geometric types of coil common used in magnetic stimulation protocols is presented. The analysis on field line behavior is highlighted, as well as the gradient estimation for each case. It is important to add the last one has been an omitted factor in magnetic stimulation research.

Keyword: Biological systems, magnetic gradients, mathematical characterization.

1 Introduction

The stimulation of biological systems using magnetic fields is a particular application where knowing the spatial distribution of the field generated by the coil plays an important role due to the need to estimate the effect of the field on the stimulated biological systems [1-3]. Typical coil geometries used in clinical, animal model, and *in vitro* stimulation systems are: single coil, Helmholtz coil, and figure eight coil arrangement.

In each case, a different effect is sought due to the characteristics of the geometry. In addition to the magnitude of the field, the systems consider some other important parameters such as the frequency of field generation, stimulation time, field homogeneity and recently giving importance to the gradient [4]. In this work the mathematical modeling using Biot-Savart’s law and geometric construction of the of the three geometries as sources of magnetic field are presented.

2 Methodology

Biot-Savart law expression describes partial contributions of the magnetic field ($d\vec{B}$) due to stationary currents (I) that flow through threadlike (or closed) circuits, considering only infinitesimal elements of length (dl), at the point located at the position that the vector ($\vec{r}$) points at a distance (r) with respect to (dl), who points in the direction of the current (I).

It is necessary to set up the vectors that represent the components according to the coil geometry. The first case to analyze is single coil geometry, where the infinitesimal element of length is given by:

$$d\overrightarrow{l} = dr\widehat{e}_r + rd\theta\widehat{e}_\theta + rsin(\theta)d\phi\widehat{e}_{\phi.} \tag{1}$$

Likewise, the vector that describes the position of any point of the field source and the observation point P is considered:

$$\vec{r} = r\widehat{e_r}, \tag{2}$$

$$\overrightarrow{r'} = r'^{\widehat{e_r}}. \tag{3}$$

Performing the corresponding vector operations, the expression that determines the field in space due to a solenoid in cartesian coordinates is:

$$B = \frac{N\mu_0 Ir}{4(r-R)^2}\left[\left(\frac{zx}{ar}\right)\widehat{i} + \left(\frac{zy}{ar}\right)\widehat{j} - \left(\frac{a}{r^2}\right)\widehat{k}\right] \tag{4}$$

Using a similar methodology as in the case of the solenoid coil, the expression of the magnetic induction for the Helmholtz coils system and the figure eight coil arrangement is obtained:

$B = \frac{N\mu_0 Ir^2}{2}\left[\frac{1}{\left(\left(z-\frac{r}{z}\right)^2+r^2\right)^{3/2}} + \frac{1}{\left(\left(z+\frac{r}{z}\right)^2+r^2\right)^{3/2}}\widehat{k}\right]$	(5)
$B = \frac{N\mu_0 Ir}{4(r-R)^2}\left[\left(\frac{z(x-a)}{ar} + \frac{z(x+a)}{ar}\right)\widehat{i} + \left(\frac{zy}{ar}\right)\widehat{j} - \left(\frac{a}{r^2}\right)\widehat{k}\right]$	(6)

A complementary parameter for stimulation systems is the gradient, this value is derived from obtaining the field at all points in space in an analytical way, this is represented by the expression:

$$\frac{d\vec{B}}{dx} = \frac{\vec{B}_2 - \vec{B}_1}{x_2 - x_1} \tag{7}$$

This methodology makes it possible to characterize the field lines generated by the three coil geometries and thus perform a calculation of the characteristic gradient for each coil.

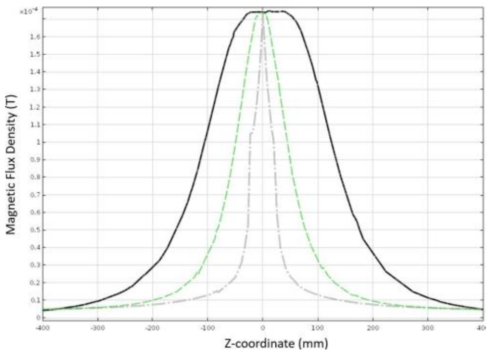

Fig. 1. Intensity of magnetic induction along the Z axis. The distribution of the Helmholtz coils system is shown in solid lines, the distribution due to the Single coil is shown in dashed lines, and the distribution due to the Figure eight coil arrangement is shown in dashed dotted lines.

3 Results

The graph of the field vectors generated by the three coils models along the Z-axis is shown in Fig. 1.

It is important to emphasize that each coil geometry has particular field generation characteristics and that they are used for different purposes, the following can be mentioned:

– Single coil: the field generated by this coil is homogeneous in the space close to the axis of symmetry of the geometry.
– Helmholtz coils system: a homogeneous field is generated in a volume that is determined by the radius of the coil. It has a field gradient close to zero.
– Figure eight coil arrangement: a focus of the field is sought with low penetration depths.

The importance of the gradient for the three cases is based on the comparison of research papers where the effects of stimulation are different, under similar stimulation conditions. The gradient values for the three models are: 1 $^{mT}/_{m}$ for the solenoid coil, 0.0001 $^{mT}/_{m}$ for the Helmholtz coil and finally 0.1 $^{mT}/_{m}$ for the figure eight coil arrangement.

4 Conclusion

The mathematical modeling through the expression of the Biot-Savart law is a core part of the work since the geometric considerations taken into account are the basis for the proposal of the model used for the simulation by the finite element method. The work carried out can be a guideline to estimate the effect of magnetic stimulation, since both the spatial distribution of the field and its direction at any point in space are known in detail.

5 Discussion

In the magnetic stimulation of cell cultures or biological systems, four parameters are identified that should be considered in each study of this nature if reproducibility is to be achieved: the intensity of magnetic induction, the frequency of magnetic induction, the length of time of magnetic stimulation and the magnetic field gradient.

Number one considers static magnetic fields, although if one considers Faraday's law of induction, where it is stated that the change of magnetic flux with time induces an e.m.f, the effect is expected (and it is in cell cultures) to be more evident.

Considering the research presented in this manuscript, it is indisputable that the gradient of magnetic induction is different in each of the three cases and therefore it is to be expected that the effect of stimulation using a coil or a system of coils in the form of eight, is greater due to Faraday's Law (considering now also the spatial difference in intensity) than that achieved with a Helmholtz coils system.

In our research group there is evidence of the effect of the magnetic field gradient, which is increased from 12% to 80% in the inhibition of the formation of protein aggregates typical of Alzheimer's disease [5].

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